Master Thesis
Economics And Public Policy
Network Interactions and Innovations

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Abstract

Sharing of knowledge, the diversity of information sources and the capacity to deal simultaneously with several fields of expertise have long been shown as catalysts to innovation. On the other hand, firms that might have an incentive to cooperate in R&D activities in order to enhance innovation strength might have different objectives on the product market. They might even be competitors.

We develop a model of competitive market à la Cournot. We add an additional stage to the optimization problem: a firm can establish collaborative links on the research side of the market. Once a link is created, there are complete spillovers of R&D efforts. Efforts in R&D are endogenized.

Due to the complexity of the mathematical framework, we simulate with Matlab the market outcomes for various competition and research collaboration matrices. We identify some recurrent patterns like the stability of only the complete and complementary collaboration networks. We then tried to investigate whether or not those recurrent patterns were mathematical features of the model. We could not conclude on this part of the problem. We hope that further researches will help disentangle the complexity of the problem formulation.

1 Introduction

Economic literature on innovation has mostly ignore the impact of network interactions between players on the dynamics of the process. These last few years, the literature on social networks has flourished and we think that considering network interactions would greatly help understand how innovation occurs within a competitive market. Indeed, all imitators cannot copy all innovators and firms that compete on the product market do not necessarily follow the same research paths. Oligopolistic competition on quantities has long been described in the literature since Cournot developed it during the XIXth century. In a simple Cournot competition, all firms have the same technology and the price is given by the total quantity of produced good. Once price is given, each firm in the market chooses its optimal quantity of good so as to optimize its profit. Introducing heterogeneity in the technology of production does not change much since the problem stays static. Therefore, we would like to enrich Cournot's framework in order to better describe how innovation happens in a competitive market. The goal of the thesis will be to use the tools of networks theory to reconsider some central questions in innovation theory. First, we will examine how the interactions between firms in the product market affects their incentives to cooperate in research. Second, we will show how viewing the absorptive capacity (Cohen et al [8]) as imposing a specific structure can affect the general lessons on the need for patent protection.

Our goal is to introduce two matrices into the framework:

- A competition matrix
- A research cooperation matrix

The former allows to describe markets in which all firms are not direct competitors. The latter allows firms to cooperate and form bilateral agreements on the R&D process. The result of those bilateral agreements is a decrease in production costs for both cooperating firms. The decrease in production cost depends on both firms's efforts in R&D. With this framework, we introduce new degrees of freedom that has not been explored yet. Our intuition is that some structural arguments on both matrices can help endogenize the research cooperation matrix and the resulting efforts in R&D.

2 Application

Characterizing fully the model we present in this master thesis would be very helpful in several fields of application. Strategic alliances are proved to be present in several industries. Burgers, Hill and Kim (1993) [9] documented it for the automobile industry and Slack, Comtois and McCalla (2002) [1] documented it for the container shipping industry. As for innovation, Cohen and Levinthal (1990) [8] described what they call the Absorptive Capacity: the capacity to exploit external knowledge is essential to innovative power. By creating common research centers, firms might catalyze their innovation capacity by making available their respective knowledge.

First, we can consider the class of public good problems:

- Step 1: invest in shared technology that reduces the cost of taking action in step 2 (sub community invests in garbage collection technology for instance)
- Step 2 : participate in public good problem (effort in keeping the neighborhood clean)

Second, we can consider the class of political economy problems:

- Step 1 : lobbying efforts that affect the cost of action
- Step 2 : investment with localized spillovers

Third, we can consider co-authorship network:

- Step 1: invest in learning a technique with subgroup
- Step 2 : effort in the interaction with co authors

Lakhani and Panetta (2007) [4] [1] explains the principles of distributed innovation. Distributed innovation happens when people who create are not acting within the same entity. It typically mimic our competition matrix. In the past, knowledge and know-how had to be located within the same entity for the innovation to finally happen. In absence of such aggregation of

[1] `http://www2.sa.unibo.it/summer/testi/18_sobrero_verganti/Principles-of-Distributed-Innovation.pdf`

knowledge or know-how, innovation would be slower and would lack possible improvements brought by someone with a different but related speciality. What the new technologies now allows is the possibility of this aggregation mechanism for a cheap price. Distributed innovation is particularly present in open source projects. In this innovation framework, developers share information, form clusters to specialize on specific technical problem, but they remain competitors. Indeed, if a new solution overcome the old one, then the creator of the old innovation is kicked off of the "market". Data on open source projects exist and are available. They can be used to test the predictions of our model. A complete analysis of Innovation on Internet and the importance of collaboration between users can be found here[2].

Competitive interactions between profit-maximizing firms in a proprietary vs. open source software industry has been studied by Casadesus-Masanell and Llanes (2010) [6]. They study the incentive of a competitive firm to open or not its software modules. The decision to open can be seen as a desire to collaborate in our framework. While Casadesus-Masanell and Llanes (2010) focus on a single firm, we introduce a matrix of competition and collaboration.

3 Empirical Data

The prediction of our model should be tested using the various databases on patents. Once the forces driving our model will have been made explicit, it should be confronted with data on common patents (patents filed by more than one firm). Another empirical validation of our model would be to take data used by Chaney [2] in order to study how an exporter reaches a new market with its product. In his framework, firms have to develop a network of importers in the targeted new markets. When a product reaches a new market, it can be seen as an innovation and therefore, we can use data gathered by Chaney.

4 Literature

This master thesis is part of the literature on networks. We present briefly few elements present in the literature that we build upon.

[2] `https://dl.dropbox.com/u/452994/Innovation_draft.pdf`

4.1 R&D and Social Networks

Goyal and Moraga-Gonzalez (2001) [7] considers that horizontally related firms can cooperate and form links. Once established, firms exert some efforts that spill over to any firms it is linked with. Efforts are costly and reduce the marginal cost of production.

Goyal and Moraga-Gonzalez (2001) concludes intuitively that in homogeneous markets (all firms are competing on the same market.), efforts are decreasing in collaborative activity. Authors also discuss the notion of stability, which is interesting since it is a matter of interest in our framework too. Authors find that the complete research network is the only stable in homogeneous markets.

Our framework improves on Goyal and Moraga-Gonzalez (2001) because we allow for any patterns of competition. Indeed, when Goyal and Moraga-Gonzalez (2001) studies competition in homogeneous markets, they only consider a particular case where **all** firms are active on the **same** product market. By introducing a competition matrix, our model could help understand many more competition patterns. For now, our competition matrix components take only values in $g_{i,j} = \{0, 1\}$, so that two firms i and j compete or not on the same market. But we can imagine an extension of the model where $g_{i,j}$ can take a continuous real positive or negative value. By doing so, we could control for the intensity of competition between two firms or allow for complement/substitute goods.

4.2 The centrality in the network as a key measure

Ballester, Calvo-Armengol, Zenou (2006) [3] is important since it links the efforts of players with a measure of their centrality in the network. That is to say, the efforts of a player depend on an idiosyncratic measure in the network. In this paper framework, the utility of each agent depends:

- Positively on his level of efforts
- Negatively on the square of his efforts
- On the interaction with every other firms

The best response is linear. The important theorem of the paper is that if the network effects are not too important, then the interior solution for each player is proportional to his Bonacich centrality[3]. The Bonacich centrality has been used in many network analysis and we want to see if it can help us understand the outcomes of our model.

A first milestone for our model would be to concentrate on interior solutions. That is why this paper by Ballester, Calvo-Armengol, Zenou (2006) is important. It gives us a closed form solution of the efforts made by agents in games where the payoffs are linear quadratic. Nevertheless, our framework is much more complicated than Ballester, Calvo-Armengol, Zenou (2006). Indeed, in their paper, utility of agents only depends on efforts (they only have one network). In our model, we have two degrees of freedom:

- Efforts in R&D
- Quantity produced

Therefore, firms' utility in our model depends only indirectly on efforts: this is because efforts reduce the cost of production that firms might have an incentive to produce more ceteris paribus.

4.3 Strategic Interaction and Networks

Bramoullé, Kranton and D'Amours (2011)[10] builds on Ballester, Calvo-Armengol, Zenou (2006) but extends the condition under which equilibria exist. Bramoullé, Kranton and D'Amours (2011) consider games where agents have linear best responses and strategic interactions are substitute (if strategic interactions are complement, then efforts evolve in the same direction and the maximum is reached).

This paper is important for us because it provides a condition under which the Nash equilibrium exists and is unique. This condition depends on the structure of the network matrix: the minimum eigenvalue of the network. If the network effect is lower than the inverse of the opposite of the lowest eigenvalue, then the unique equilibrium is interior and it is the Nash equilibrium. The intuition is that when the graph is sufficiently tight, then there is a unique equilibrium. When the graph is loose, equilibria might be multiple and stable ones always involve extreme plays by the players.

We use this condition in our last stage optimization on quantities. By doing so, we concentrate our analysis on interior solutions. Bramoullé, Kranton

[3]The Bonacich centrality is a measure that keeps track of all the paths of any length in the network that starts with a particular node.

and D'Amours (2011) gives us a condition on the structure on the competition matrix in order to adjust the degree of the impact of firm j on selling prices of firm i.

5 The Model

Here is an example on how elements of network interactions can be included into a model of competitive markets. We focus on interior solutions where any firms exert some positive efforts in R&D and produce some positive quantities of good. The problem is solved in backward induction. We first solve the last stage during which firms optimize profits with respect to quantities. Then, we solve for the optimal profit with respect to efforts in R&D.

5.1 Profit Optimization on quantities

Let N be the set of firms. The competition matrix is given by the coefficients $[g_{ij}], \forall i, j$, with $g_{ii} = 0$. a and b are constants. δ is the magnitude of the competition effect. Profit is given by:

$$\pi_j = (a - bq_j - 2\delta b \sum_{l \neq j} g_{jl} q_l) q_j - c_j q_j \tag{1}$$

This supposes that the price is given by:

$$p_j = (a - bq_j - 2\delta b \sum_{l \neq j} g_{jl} q_l)$$

We see that depending on the nature of the competition, the effect of quantities on prices is not the same. For instance, if all firms operate on the same exact product market, then all $g_{jl} = 1$, $\forall j \neq l$ and the impact of one more unit of production on the price is the same regardless of the firm that produced the good. If, on the contrary, firms operate on more distant product markets, then we see that the more distance between firms j and l (low g_{jl}), the less the impact of quantity q_l on p_j.

We derive profit π_j with respect to q_j. With $g_{jj} = 0$, we get:

$$q_j^* = \frac{a - c_j}{2b} - \delta \sum_{l \neq j} g_{jl} q_l \tag{2}$$

It has to be true for any firm j that produces a positive quantity. Otherwise, $q_j^* = 0$. Thus, q_j^* is given by expression (2) if it yields a positive number, and 0 otherwise. As said above, we concentrate on interior solutions where all firms are active (producing a positive quantity). We normalize $b = 1$. We note $\boldsymbol{G}_j$, the j-th line of matrix $\boldsymbol{G}$, $\boldsymbol{q}$ the vector of quantities and $\mathbb{1}$ the vector of ones. We get:

$$\begin{aligned} q_j^* &= \frac{a - c_j}{2} - \delta \boldsymbol{G}_j \boldsymbol{q}^* \\ \boldsymbol{q}^* &= (\boldsymbol{I} + \delta \boldsymbol{G})^{-1} \frac{a\mathbb{1} - \boldsymbol{c}}{2} \end{aligned} \tag{3}$$

We plug the expression of q_j^* back into the profit expression (1). We get:

$$\pi_j = q_j^2 \tag{4}$$

5.2 Investment choices

We note $\boldsymbol{H}$ the research network matrix, $\boldsymbol{H}_j$ the j-th line of $\boldsymbol{H}$, d a constant and $\boldsymbol{x}$ the vector of efforts. The problem is the following:

$$\begin{aligned} &max\, \pi_j - dx_j^2 \\ &s.t.\, c_j = c_0 - H_j \boldsymbol{x} \end{aligned}$$

We derive with respect to effort x_j. We get:

$$2dx_j = \frac{\partial \pi_j}{\partial x_j} \tag{5}$$

With expression (4) of profit at equilibrium, we get:

$$\frac{\partial \pi_j}{\partial x_j} = 2q_j^* \frac{\partial q_j^*}{\partial x_j} \tag{6}$$

We go back to the expression of the equilibrium quantities for active firms. We note $\boldsymbol{D} = (\boldsymbol{I} + \delta \boldsymbol{G})^{-1}$. We have:

$$q_j^* = \boldsymbol{D}_j \frac{(a - c_0)\mathbb{1} + \boldsymbol{H}\boldsymbol{x}}{2}$$

After some algebra we get:

$$\boldsymbol{D}_j\left[\boldsymbol{H}\boldsymbol{x}\right] = d_{j,1}\sum_i h_{1,i}x_i + d_{j,2}\sum_i h_{2,i}x_i + \cdots + d_{j,n}\sum_i h_{m,i}x_i$$

We note $\boldsymbol{H}^j$ the j-th column of matrix $\boldsymbol{H}$, it yields:

$$\frac{\partial q_j}{\partial x_j} = \frac{1}{2}\boldsymbol{D}_j\boldsymbol{H}^j$$

Now, we go back to expression (5). After some algebraic re-arrangments, we have the following closed form solution for the efforts:

$$\boldsymbol{x} = (a - c_0)\left(4d\cdot\boldsymbol{I} - \begin{bmatrix}\boldsymbol{D}_1\boldsymbol{H}^1\boldsymbol{D}_1\boldsymbol{H}\\ \boldsymbol{D}_2\boldsymbol{H}^2\boldsymbol{D}_2\boldsymbol{H}\\ \vdots\\ \boldsymbol{D}_n\boldsymbol{H}^n\boldsymbol{D}_n\boldsymbol{H}\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}\boldsymbol{D}_1\mathbb{1}\boldsymbol{D}_1\boldsymbol{H}^1\\ \boldsymbol{D}_2\mathbb{1}\boldsymbol{D}_2\boldsymbol{H}^2\\ \vdots\\ \boldsymbol{D}_n\mathbb{1}\boldsymbol{D}_n\boldsymbol{H}^n\end{bmatrix} \tag{7}$$

6 Research

6.1 General framework

Our goal is to find some conditions on the structure of both $\boldsymbol{G}$ and $\boldsymbol{H}$ that ensure stability of $\boldsymbol{H}$. Indeed, the competition matrix might be considered exogenous. But once it is observed, firms decide on what the collaboration matrix is. We define stability similarly to the usual practice in the networks literature. Let h be the collaboration network associated with the matrix $\boldsymbol{H}$.

Stability A collaboration network h is stable if and only if $\forall i, j \in N$,

- (i) if $h_{i,j} = 1$, then $\pi_i(h) \geq \pi_i(h - h_{i,j})$ **and** $\pi_j(h) \geq \pi_j(h - h_{i,j})$
- (ii) if $h_{i,j} = 0$, then $\pi_i(h + h_{i,j}) < \pi_i(h)$ **or** $\pi_j(h + h_{i,j}) < \pi_j(h)$

We also would like to analyze total efforts and total profits of all the firms. The idea is to have a measure of what would be desirable from the point of view of the overall industry/society.

Our intuition is that the incentives for a firm to create or delete collaboration links is related to the profile of competition faced by a particular firm.

6.2 Research Strategy

The first step for us has been to verify the point made by Goyal and Moraga-Gonzalez (2001) that in homogeneous good competition markets (complete matrix $\boldsymbol{G}$), then the only stable matrix for the research network is the complete matrix.

The final expression (7) for the efforts is complicated at first sight and it is difficult either to say anything on the shape of the solutions or to link it to the structure of both $\boldsymbol{G}$ and $\boldsymbol{H}$.

Therefore, we decided to implement the solution on Matlab and to simulate market outcomes (efforts, profits and quantities) for any interesting research networks $\boldsymbol{H}$. For the parameter δ, we followed Bramoullé, Kranton and D'Amours (2011) and decided to take half of the inverse of the opposite of the lowest eigenvalue of $\boldsymbol{G}$. The impacts of tougher competition links and the subsequent non-interior solutions are to be investigated in later researches. The source code has been written in such a way that it is easily modifiable. The source code should be working for any nxn firms framework. The problem is only the computation time. Indeed, for a nxn matrix, there are $K = \frac{n(n-1)}{2}$ possible links and 2^K possible networks. Then, for any network $\boldsymbol{H}$, there are K adjacent matrices [4] that should be tested for stability.

As a result, we simulated all cases for 3x3 matrices in order to compare our results to Goyal and Moraga-Gonzalez (2001) when the competition matrix is complete. Then, we simulated all possible $\boldsymbol{H}$ when the competition matrix is regular [5]. Finally, we simulated any other interesting competition matrix that could help us in identifying properties of our model.

An archive with the MatLab source code can be found here: `https://dl.dropbox.com/u/452994/n_firms_code.zip`.

6.3 Simulation results

6.3.1 Complete Competition Market

We indeed confirm Goyal and Moraga-Gonzalez (2001)'s point that with the complete competition matrix, the only stable research network is the complete one. We tested it for 3x3, 4x4, 5x5, and 6x6. We can give the intuition

[4] matrix that differs only by one link

[5] matrix in which all firms have the same number of links

of the economic forces driving this result by presenting the 3x3 case.

Starting with the empty research network, firms 2 and 3 have an incentive to create a link because it reduces both their production costs and as a result gives them a comparative advantage with regards to firm 1. Both firms 2 and 3 decrease their respective efforts in R&D once the link is created. The intuition is that when a firm creates a link with a competitor, it has an incentive to free-ride on the partner's efforts. Nevertheless, the decrease of efforts by both firms does not offset the benefit from the spillover of partner's efforts in reducing production costs.

Then, a link is created between firms 1 and 2. Indeed, as all firms are competing on the **same** product market, firm 1 has an incentive to decrease its production costs and firm 2 is still willing to accept because it gives it a competitive advantage with respect to firm 1 and 3. Again, the formation of link between firms 1 and 2 reduces their respective efforts in R&D; but it is more then offset by the production cost reduction force.

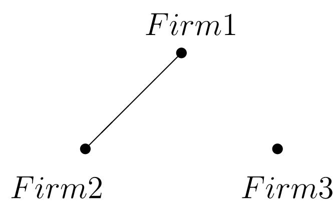
Finally, still because all firms are symmetric on the competition market, firms 1 and 3 create a research link because it increases their profit to the same level as firm 2.

We also observe that the complete research network is the one in which firms make the less efforts. Indeed, as the competition network is complete, all efforts in R&D are substitutes and firms have an incentive to form collaboration links and to free-ride on partners' efforts. As the game is a strategic one, all firms know the others' incentive to free-ride. As a result, firms create links because it reduces costs, but they also decrease their efforts.

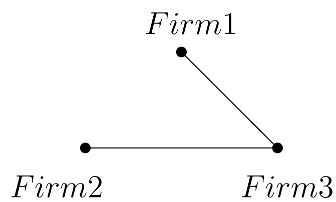
This is just an example for the complete competition network with three firms. We are curious of how the forces at work that we just introduced would interact in more complex competition frameworks. In order to have a clear picture of the underlying mechanisms, we keep on with the presentation of strategic interactions with 3 firms. We then extrapolate and investigate if we can generalize our results to any nxn networks.

6.3.2 The partially connected competition matrix

The matrix $\boldsymbol{G}$ is:



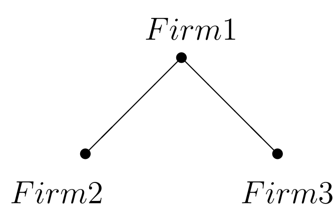
In this case, the only stable research network is: $\boldsymbol{H} =$



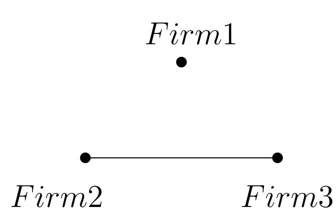
Why the complete research network is not stable? Because, if firms 1 and 2 create a research link, both decrease their effort in R&D because they know that they have an incentive to free-ride on each other. The spillover from the partner firm is not enough to offset the cost production increase due to efforts lowering. On its part, firm 3 also decreases its effort because it knows that the more links its partners have with competitive firms, the more incentive they have to free-ride and lower their efforts in R&D. Therefore firm 3 adjusts its own efforts too. Ultimately, the collaboration link between firms 1 and 2 is not created.

6.3.3 The star competition network with firm 1 as the hub firm

The matrix $\boldsymbol{G}$ is:



In this case, the only stable research network is:



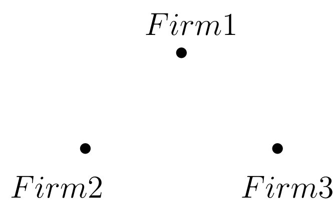
The forces at work in this particular case are the same as in the previous example with the partially connected competition network. Starting with the empty collaboration network, firms 2 and 3 have an incentive to create a link because their efforts are substitute: they are not competing on the product market so their in R&D's efforts increases with the creation of this link.

On the other hand, starting from the empty collaboration network, firms 1 and 2 do not have an incentive to create a collaborative link. As they are competitors on the product market, both decrease their efforts in R&D with the link creation. Firm 1 still increases its profits with the formation of link 1-2 because it is competing also with firm 3 which has no collaboration link. As a result, firm 1 would like to create that link because it gives it a competitive advantage with respect to firm 3. As firms 1 and 3 are competing on the same market, if firm 1 can reduce its production cost while firm 3 stays with high production costs, then firm 1 gets a competitive advantage. Nevertheless, in order for the link to be created, firm 2 has to agree and we see in the simulation results that it is not the case. The profits of firm 2 decrease with the creation of the collaboration link 1-2. The reason is that firm 1 is its only competitor so it has no additional incentive to gain some competitive advantage with respect to a hypothetical competitive firm.

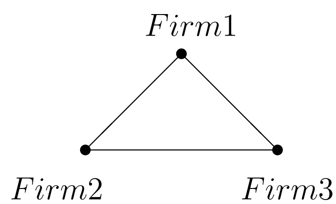
Now, starting with the collaboration network where firms 2 and 3 have a collaboration link, we explain why there is no incentives to create any additional links. Firms 1 and 3 could create a collaboration link. Firm 1 would make more profits and lower its efforts in R&D because it would benefit from the high level of effort of firm 3 due to its collaboration with (substitute) firm 2. Moreover, the creation of this link 1-3 would lower firm 3's efforts and this would harm its research collaborator firm 2. Nevertheless, firm 3 is not willing to create this link. It already has a link with firm 2 which is a substitute firm. Therefore, its efforts and profits are high. Creating a link with firm 1, which is a competitor, would enable it to free-ride on its high R&D efforts. As firm 1 has no additional link with substitute firms, firms 3 knows that firm 1 would **only** free-ride its efforts. As a result, the link 1-3 is not created and the only stable research network is the complementary network where only firms 2 and 3 share R&D efforts and knowledge.

6.3.4 The empty competition network

The matrix $\boldsymbol{G}$ is:



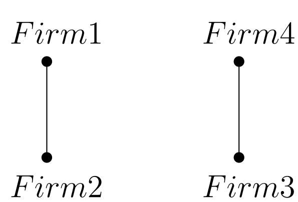
In this case, the only stable research network is:



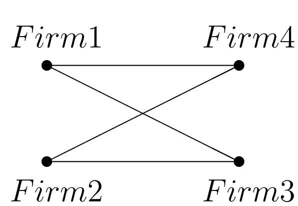
For the empty competition network, it is easily understandable that the only stable collaboration network is the complete one. Indeed, all efforts are substitute and all firms have an incentive to have the highest number of collaboration links.

6.3.5 The complementary networks

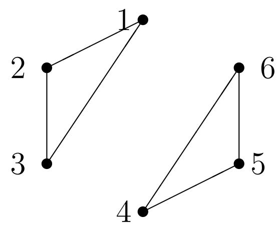
Looking at all the previously presented cases, we see that the only stable research networks are either the complete one and the complementary one. Our intuition lead us to think that the complementary competition and research networks could be stable in cases where the competition matrix separates firms into several subgroups. For instance, in the 4x4 firms framework, the competition matrix $\boldsymbol{G}$=



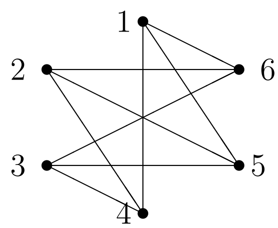
yields the complementary research network as the only stable one, $\boldsymbol{H}$=



But there seems to have other forces at play since the similar case in the 6x6 firms framework yields a different result. Indeed, the following competition matrix, $\boldsymbol{G}$=

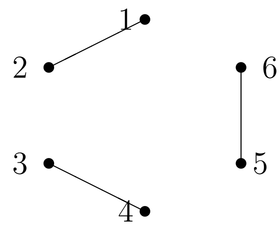


yields the complete research network as the only stable. Our intuition would have been that the following research network is stable, but it is not, $\boldsymbol{H_{C_1}}$=

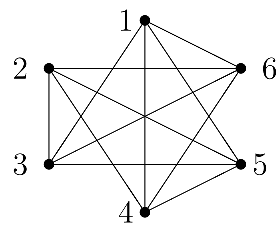


We do not have the details yet, but maybe starting with the research network $\boldsymbol{H_{C_1}}$ presented above, firm 1 has an incentive to create a link with firm 2 and firm 3 who would retaliate by creating link 3-2 and so on until the complete collaboration network is reached. The rationale behind this could be that starting with the research network $\boldsymbol{H_{C_1}}$, the efforts are so high that the decrease in efforts induced by a new link with a competitor does not create a sufficient production cost increase to offset the spillover. It would also mean that the marginal effect of links on efforts depends on the structure of matrix $\boldsymbol{G}$. It would also mean that the more competitors are active on the **same** product market , the more likely it is that the complete research network would be stable. Maybe there exists a certain threshold above which firms can create links with competitors because the efforts are already high enough to offset negative spillovers.

This intuition is confirmed by the following example in which the structure of the competition market is exactly the same except that there are 2 firms competing in each subnetwork instead of 3 in the previous example. The competition network $\boldsymbol{G}$=



yields the complementary research network as the only stable, $\boldsymbol{H_{C_2}}=$



Contrary to the case in which the competition matrix is composed of 2 complete subnetworks of three firms, the one with 3 complete subnetwork of two firms yields the complementary collaboration network as the only stable. It is in accordance with our intuition but we are not able to prove it mathematically.

6.4 Properties of the Model

6.4.1 Instability of the empty research network

In all the simulations that we performed, we could not find any competition networks that would yield the empty research network as stable. Our intuition is that it is a general feature of the model, so we tried to show it mathematically, but we could not achieve that goal. Indeed, the expression of efforts given by expression (7) is so complicated that we could not conclude.

We note t_i the Bonacich centrality of firm i in the competition network $\boldsymbol{G}$. We note $\boldsymbol{T}$ the vector of Bonacich centralities. We have $\boldsymbol{T} = (\boldsymbol{I} + \delta\boldsymbol{G})^{-1}\boldsymbol{G}\mathbb{1}$. Following Bramoullé, Kranton and D'Amours (2011), we have:

$$\begin{aligned}(\boldsymbol{I} + \delta\boldsymbol{G})(\boldsymbol{I} + \delta\boldsymbol{G})^{-1}\mathbb{1} &= \mathbb{1}\\ (\boldsymbol{I} + \delta\boldsymbol{G})^{-1}\mathbb{1} + \delta\boldsymbol{G}(\boldsymbol{I} + \delta\boldsymbol{G})^{-1}\mathbb{1} &= \mathbb{1}\\ \boldsymbol{D}\mathbb{1} + \delta\boldsymbol{T} &= \mathbb{1}\end{aligned}$$

If the research network is empty, then $\boldsymbol{H} = \boldsymbol{I}$, and we have:

$$\begin{aligned} \boldsymbol{D}_i \boldsymbol{H}^i &= d_{i,i} \\ \boldsymbol{D}_i \boldsymbol{H} &= \boldsymbol{D}_i \\ \boldsymbol{D}_i \mathbb{1} &= 1 - \delta t_i \end{aligned}$$

We report into expression (7), we get

$$\boldsymbol{x} = (a - c_0) \left(4d \cdot \boldsymbol{I} - \begin{bmatrix} d_{1,1}\boldsymbol{D}_1 \\ d_{2,2}\boldsymbol{D}_2 \\ \vdots \\ d_{n,n}\boldsymbol{D}_n \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} (1-\delta t_1)d_{1,1} \\ (1-\delta t_2)d_{2,2} \\ \vdots \\ (1-\delta t_n)d_{n,n} \end{bmatrix} \tag{8}$$

The matrix

$$\begin{bmatrix} d_{1,1}\boldsymbol{D}_1 \\ d_{2,2}\boldsymbol{D}_2 \\ \vdots \\ d_{n,n}\boldsymbol{D}_n \end{bmatrix}$$

has unknown rank. It can go from rank 1 to full rank depending on the structure of the matrix $\boldsymbol{G}$. Therefore, we cannot have a simple closed form solution for efforts, quantities and profits.

6.4.2 Stability of the complete research network

Next, we tried to understand the underlying reasons behind the stability of the complete research network. As we have seen previously, it seems that both the complete and the complementary networks are the only stable ones. But what could make the complete network unstable? We tried to investigate this question by expressing the efforts, quantities and profits first under the complete research network and then when we remove a collaborative link. The idea is that if for some reasons the complete research network is not stable, then there exists a link that can be deleted and it would lead to higher profits for at least one of the firms involved.

6.4.2.1 A world of complete R&D spillovers, the complete research network

When the research network is complete, the matrix $\boldsymbol{H}$ is composed only of 1. Then, we have:

$$\begin{aligned}\boldsymbol{D}_i\boldsymbol{H}^i &= \sum_j d_{i,j} = 1-\delta t_i\\ \boldsymbol{D}_i\mathbb{1} &= 1-\delta t_i\\ \boldsymbol{D}_i\boldsymbol{H} &= (1-\delta t_i)\,[1\ldots 1]\end{aligned}$$

The expression of efforts is then given by:

$$\boldsymbol{x} = (a-c_0)\left(4d\cdot\boldsymbol{I} - \begin{bmatrix}(1-\delta t_1)^2\,[1\ldots1]\\ (1-\delta t_2)^2\,[1\ldots1]\\ \vdots\\ (1-\delta t_n)^2\,[1\ldots1]\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}(1-\delta t_1)^2\\ (1-\delta t_2)^2\\ \vdots\\ (1-\delta t_n)^2\end{bmatrix} \tag{9}$$

We see that the second operand in the inverse matrix is of rank one. We use Miller (1981) [5] in order to find the inverse of the matrix involved in expression (9). Miller (1981)'s key result is that if $\boldsymbol{B}$ has rank 1, then

$$(\boldsymbol{A}+\boldsymbol{B})^{-1} = \boldsymbol{A}^{-1} - \frac{1}{1+u}\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}^{-1} \tag{10}$$

with $u = trace(\boldsymbol{B}\boldsymbol{A}^{-1})$.

We set $A = 4d\boldsymbol{I}$ and $B = -\begin{bmatrix}(1-\delta t_1)^2\,[1\ldots1]\\ (1-\delta t_2)^2\,[1\ldots1]\\ \vdots\\ (1-\delta t_n)^2\,[1\ldots1]\end{bmatrix}$. We finally get the expression for the efforts:

$$\boldsymbol{x} = \frac{a-c_0}{4d-\sum_j(1-\delta t_j)^2}\begin{bmatrix}(1-\delta t_1)^2\\ (1-\delta t_2)^2\\ \vdots\\ (1-\delta t_n)^2\end{bmatrix} \tag{11}$$

We now express the quantities produced. We start with expression (3) and we incorporate the final expression of efforts (11). We use the fact that

$$\boldsymbol{DH} = \begin{bmatrix} (1-\delta t_1)^2 \, [1 \dots 1] \\ (1-\delta t_2)^2 \, [1 \dots 1] \\ \vdots \\ (1-\delta t_n)^2 \, [1 \dots 1] \end{bmatrix}$$

We get:

$$\boldsymbol{q} = \frac{2d(a-c_0)}{4d - \sum_j (1-\delta t_j)^2} \begin{bmatrix} (1-\delta t_1) \\ (1-\delta t_2) \\ \vdots \\ (1-\delta t_n) \end{bmatrix} \tag{12}$$

Profits are just given by taking the square of individual quantities. We get:

$$\boldsymbol{\pi} = \frac{4d^2(a-c_0)^2}{(4d - \sum_j (1-\delta t_j)^2)^2} \begin{bmatrix} (1-\delta t_1)^2 \\ (1-\delta t_2)^2 \\ \vdots \\ (1-\delta t_n)^2 \end{bmatrix} \tag{13}$$

First we discuss the behavior on the efforts side. The denominator of the expression (11) is the same for all firms. As the overall centrality (captured by the term $\sum_j (1-\delta t_j)^2$) increases, the overall efforts decrease. The intuition is that in a world of complete spillovers on the R&D side, the sum of efforts is high when firms operate on different markets. If the competition network is tight, then firms do not want to share their R&D efforts.

Each firm is also affected by an idiosyncratic term depending on its own centrality in the competition market. Intuitively, the more central a firm is, the less efforts exerted as compared to firms with a lower competition environment. It has to be noted that produced quantities are less affected than efforts by own centrality.

We also note that the ratio $\frac{Profit}{Efforts}$ is the same for all firms and it does not depend on idiosyncratic terms. We have:

$$\frac{\pi_j}{x_j} = \frac{4d^2(a-c_0)}{4d - \sum_j (1-\delta t_j)^2}, \forall j \tag{14}$$

It means that if two firms create a link on the competition network, all firms will adjust their efforts so as to equalize their "efforts profitability" with every other firms. Nevertheless, we see that the "efforts profitability" decreases with own centrality as well as with the centrality of research partners.

Another interesting feature is the negativity of the derivative of profits with respect to own centrality. Negativity of the derivative is ensured by the positivity of efforts and profits. The positivity of quantity is ensured by the condition from Bramoullé, Kranton and D'Amours (2011) on the lowest eigenvalue of $\boldsymbol{G}$.

$$\frac{\partial \pi_j}{\partial t_j} = \frac{-2\delta(1-\delta t_j)(4d - \sum_i (1-\delta t_i)^2)^2 - 4\delta(1-\delta t_j)^3(4d - \sum_i (1-\delta t_i)^2)}{(4d - \sum_i (1-\delta t_i)^2)^4} \tag{15}$$

What does this mean? That for a constant number of firms and a world of complete spillovers on the R&D side, then there is no firm with incentives to increase its centrality by creating competition links. It means that a particular firm has no incentives to compete on a additional existing market.

How to model Innovation? When an innovation occurs, a new firm is created. The research network is still the complete network with full spillover of each firms' R&D efforts. The competition network is still the same, except that we have one more firm that is isolated from all other firms (when an innovation is introduced, the firm that commercializes it is the only one on its market). As the new firm is isolated on the competition network, the centralities of the already existing firms do not change. Let $n+1$ be the index of the innovative firm. $\forall i \in \{1 \dots n\}$, we have $d_{i,n+1} = 0$. Therefore, as we have $\boldsymbol{D}_i \mathbb{1} = 1 - \delta t_i$, then the centralities of existing firms stay the same.

The "profitability of efforts" $\frac{\pi_j}{x_j}$ increases following the entry of a new firm. Indeed, the sum in the denominator of expression (14) stays exactly the same except that we have an additional positive term. As a result, the ratio increases for all firms: the "old" ones and the new innovative one.

6.4.2.2 A world of <u>almost</u> complete R&D spillovers, by removing one link in the complete research network

In order to study the effect of removing one link in the collaboration network on the stability, we tried to analyze the outcomes of our model when we remove the collaboration link between firms 1 and 2. We use the following results:

$$
\begin{aligned}
\boldsymbol{D}_1\boldsymbol{H}^1 &= 1-\delta t_1 - d_{1,2}\\
\boldsymbol{D}_2\boldsymbol{H}^2 &= 1-\delta t_2 - d_{2,1}\\
\boldsymbol{D}_i\boldsymbol{H}^1 &= 1-\delta t_i - d_{i,2}\\
\boldsymbol{D}_i\boldsymbol{H}^2 &= 1-\delta t_i - d_{i,1}\\
\boldsymbol{D}_i\boldsymbol{H}^i &= 1-\delta t_i, \forall i \neq 1,2\\
\boldsymbol{D}_i\mathbb{1} &= 1-\delta t_i
\end{aligned}
$$

Then we have for the efforts the following expression:

$$
\boldsymbol{x} = (a-c_0)\left(4d\cdot\boldsymbol{I} - \begin{bmatrix}
(1-\delta t_1 - d_{1,2})\,[1-\delta t_1 - d_{1,2} & 1-\delta t_1 - d_{1,1} & 1-\delta t_1 \ldots 1-\delta t_1]\\
(1-\delta t_2 - d_{2,1})\,[1-\delta t_2 - d_{2,2} & 1-\delta t_2 - d_{2,1} & 1-\delta t_2 \ldots 1-\delta t_2]\\
(1-\delta t_3)\quad [1-\delta t_3 - d_{3,2} & 1-\delta t_3 - d_{3,1} & 1-\delta t_3 \ldots 1-\delta t_3]\\
 & \vdots & \\
(1-\delta t_n)\quad [1-\delta t_n - d_{n,2} & 1-\delta t_n - d_{n,1} & 1-\delta t_n \ldots 1-\delta t_n]
\end{bmatrix}\right)^{-1} \tag{16}
$$

$$
\cdot \begin{bmatrix}
(1-\delta t_1)(1-\delta t_1 - d_{1,2})\\
(1-\delta t_2)(1-\delta t_2 - d_{2,1})\\
(1-\delta t_3)(1-\delta t_3)\\
\vdots\\
(1-\delta t_n)(1-\delta t_n)
\end{bmatrix} \tag{17}
$$

The second operand in the matrix to be inverted is of rank 3. We could use Miller (1981) recursively in order to invert the matrix. Nevertheless, we tried to do it and the expression is way too complicated for it to be interpretable.

As a result, we are not able to give any interpretations on the effect of removing one collaboration link on the market's outcomes. Our hopes were that we could conclude on the structure of the competition matrix that would allow for the stability of the full collaboration network. Indeed, according to our simulations, only the complete or the complementary research networks are stable. By understanding the effects of removing one collaborative link, we could have found a structural condition on the competition matrix that would explain why certain competition networks yield the complete research network and some others yield the complementary one.

6.5 Problems and further research

It seems that the interaction between the competition matrix and the collaboration matrix is more complicated than we originally thought. The simulation allowed us to witness some recurrent patterns like the stability only of the complete and complementary research networks.

On the other hand, it seems like our mathematical framework to too complicated in order to solve and understand the driving forces behind the market outcomes. Indeed, at this stage, we do not know if the emergence of the complete and complementary research networks as the only stable ones is:

- The fruit of hazard
- Due to some values that we chose for the simulation
- A real feature of the model

What has been presented in this master thesis is only an example of what can be done when introducing elements of network theory into a competitive market. We see that it changes greatly the incentive of innovation. We see that there are a lot a further interesting investigations to be made in order to understand fully the interactions between the incentives to create/delete links in the research network.

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